Math 128a – Week 3 Worksheet
GSI: Izak, (2/3/21)

2.1 Problems

Problem 1. *Use the bisection method to find a solution accurate to within* 10^{-5} *of* $e^x - x^2 + 3x - 2 = 0$ *for* $0 \le x \le 1$

Problem 2. *Use the bisection method to find the intersections of* $x^2 - 1$ *and* e^{1-x^2} *accurate to within* 10^{-3}.

Problem 3. *Determine a continuous function such that the bisection method fails to converge to a zero.*

2.2 Problems

Problem 4. *Approximately solve* $f(x) = x^3 - 2x + 1 = 0$ *using a fixed point algorithm by computing* p_1, p_2, p_3, p_4 *for each method:*

1. $x = \frac{1}{2}(x^3 + 1)$ $p_0 = 1/2$
2. $x = 2/x - 1/x^2$ $p_0 = 1/2$
3. $x = \sqrt{2 - 1/x}$, $p_0 = 1/2$
4. $x = -(1 - 2x)^{1/3}$, $p_0 = 1/2$

which methods seem to be appropriate.

Problem 5. *Use a fixed-point iteration method to find an approximation to* $(25)^{1/3}$ *that is accurate to within* 10^{-4}

Problem 6. *Show that if* $g \in C([a, b])$ *and* $g(x) \in [a, b]$ *for* $x \in [a, b]$ *and* g' *exists in* (a, b) *and a constant* $k \in (0, 1)$ *exists with* $g'(x) \le k$ *for all* $x \in (a, b)$, *then it is not always true that for all* $p_0 \in [a, b]$ *the sequence* $p_n = g(p_{n-1})$ *converges to a fixed point of* g. *(hint* $1 - x^2$ *on* $[0, 1]$*)*